Operations with Integers

Definition - Absolute Value: the number value after the sign in front of it is

Example:
$$|-5| = 5$$
 $|5| = 5$

$$|5| = 5$$

Example:
$$-5 + 3 = -2$$

Example:
$$4 - (-3) = 7$$

Example:
$$-6 \times -2 = 12$$
 $3 \times -14 = -42$

$$3 \times -14 = -42$$

Example:
$$-25 \div 5 = -5$$
 $-48 \div -6 = 8$

$$-48 \div -6 = 8$$

Directions: Complete the problems below using the correct operation.

Resource Used: Bleau, Barbara Lee. Forgotten Algebra: a Self-teaching Refresher Course. Hauppauge, NY: Barron's Educational Series, 1994. Print.

Operations with Fractions

When adding or subtracting fractions, they need to have the SAME denominator! In this case, find the least common denominator. Find a number that both 5 and 10 have in common, which is 10. Multiply the numerator and the denominator by 2 in the first fraction. Your fraction changes from 2/5 to 4/10.

$$\frac{2}{5} + \frac{3}{10} = \frac{4}{10} + \frac{3}{10} = \frac{7}{10}$$

Once the denominator is the same, then you add the numerators together to get your answer. When adding and subtracting, the denominator stays the same and only the numerator changes.

Directions: Complete the following problems below using the correct operation.

1.
$$3 - \frac{5}{12} =$$

2.
$$1\frac{4}{6} + 5\frac{2}{7} =$$

3.
$$\frac{12-4(3)}{8\times(2\div1)} =$$

4. There are 20 students on the soccer team. $\frac{1}{5}$ of the students are unable to make it to the first game. How many students will be at the first game?

Operations with Decimals

Example: Solve the problems below

B.
$$3.80 \div 1.25 = ____3.04___$$

you're done!

$$380 \div 125 = 3.04$$

Directions: Solve the problems below. You must SHOW YOUR WORK!

Positive Integer Exponents, Square Roots, Radicals

 $1.4 \times 10^4 = 14.000$ Move the decimal place over 4 spaces in 1.4 based on the exponent of the number with a base value of 10.

$$2^3 = 2 \times 2 \times 2 = 8$$

$$a^{2}(2a^{3}) = 2a^{5}$$
 $(a^{2})^{3} = a^{6}$

$$(a^2)^3 = a^6$$

When multiplying exponents with the same base, add the exponents together. When raising an exponent to an exponent, multiply the exponents together.

1.
$$x^2(x^3)^4 =$$

2.
$$(3x^5)^2 (2x^3)^3 =$$

3. What is 4 x 4 x 4 as a power of 2? _____

4. There is a square with 22 inch sides. The diagonal was calculated to be $\sqrt{968}$ so what is the length of the diagonal rounded to the nearest whole number?

0

Percentages

A percent is a number out of 100. Any percent can be written as a fraction as the number given over 100. Ex. $35\% = \frac{35}{100} = 0.35$

Example: There are 150 seagulls on the beach. 16% of them have black spots on them. How many have black spots?

Set up an equation: $.16 \times 150 = 24$ seagulls

Directions: Solve the problems below using percentages.

- 1. What is 5% of 25% of 150,000? _____
- 2. Books are being sold to the bookstore \$50.00 each. The bookstore puts a 31% markup on the price of the books to sell to students. How much would a student have to pay for the book in the bookstore?

- 3. There were 50 M&M's in a bag. The first child ate 20% of the M&M's. Then a second child ate 25% of the remaining M&M's. How many were left?
- 4. 36 is what percentage of 144?

Averages (Mean, Median, and Mode)

Example: The set of scores below is from a test administered in a classroom:

To find the "mean," add up all of the scores together:

Mode is the number that occurs the most in the set: 74 ← Mode

To find the median, put the numbers in chronological order: 52, 74, 74, 88, 97

Now find which number is in the middle of the set: 74 = Median

Now try it on your own. Find the mean, median, and mode for the following sets of numbers.

1. 4, 5, 2, 4, 1

Mean:_____

Median: _____

Mode: _____

2. 19, 31, 7, 24, 14, 31, 19

Mean: _____

Median: _____

Mode:

3. 95, 94, 94, 94, 89, 90, 96, 99

Mean: _____

Median: _____

Mode: _____

4. 2, 1, 1, 1, 1, 1, 2, 1, 3

Mean: _____

Median:

Mode: _____

5. 13, 15, 20, 22, 21, 14, 17, 12, 20, 14, 15, 20

Mean: _____

Median:

Mode: _____

6. 78, 89, 54, 12, 99, 70, 81, 65, 91, 82, 88, 89, 74

Mean: _____

Median:

Mode: _____

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Substituting Values into Algebraic Expressions

Must start with PEMDAS: Parenthesis, Exponents, Multiplication/Division, Addition/Subtraction. Simplify in that order and always start with the innermost parenthesis. Then combine like terms. Don't forget to distribute your negative!

second set of parenthesis

$$3x+5xy+2y-4+3xy+2x$$
 Now combine like terms

$$5x + 8xy + 2y - 4$$
 Answer!

Directions: Simplify the following expressions. SHOW YOUR WORK.

1.
$$(7x-2y+5)+(2x+5y-4)=$$

2.
$$(5y - 2a + 1) - (y - 3a - 7) =$$

3.
$$5-2(x+2[3+x]) =$$

4.
$$-3a - 5\{2a - 2[2a - (4a - 2) - 2(7 - 3a)]\} =$$

Setting Up Equations for Given Situation

Change words into symbols: Three times a number minus 2 equals 10.

This will be changed into : 3x - 2 = 10

The **bolded** words are your operations

Five times a number plus 2 is 10: 5x + 2 = 10

$$5x + 2 = 10$$

Look for the cue words in your statement to figure out how to change the words into an equation.

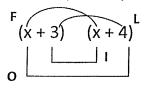
Directions: Use "x" for "a number" in the following set of sentences. Write an equation that matches up with each statement.

- 1. A number added to 20 is the same as 32.
- 2. Two times a number decreased by 5 equals 11.
- 3. The sum of x and 12 is 20.
- 4. Fifteen increased by 2 times a number is 47.
- 5. Four more than three times a number is 17.
- 6. Now try a word problem: George is 8 years older than Jack. The sum of their ages is 42. How old is each person?

Multiplication of Polynomials



FOIL: First, Outside, Inside, Last



$$F \qquad \text{OI} \qquad L$$
$$= x^2 + 7x + 12$$

Step 1. First:
$$x \cdot x = x^2$$

2. Outside:
$$x \cdot 4 = 4x$$
 3. Inside: $x \cdot 3 = 3x$

3. Inside:
$$x \cdot 3 = 3x$$

4. Last:
$$4 \cdot 3 = 12$$

Combine your outside and inside terms:
$$4x + 3x = 7x$$

Directions: Using FOIL, complete the following problems.

1.
$$(x-7)(x-2) =$$

2.
$$(2x + 3)(x - 1) =$$

3.
$$(3x + 1)(2x - 5) =$$

4.
$$(x + 2y) (x - 3y) =$$

5.
$$(x^2 + 5) (x - 3) =$$

6.
$$(5x-2)(x+2) =$$

Factoring Polynomials

When factoring, make sure to first look for a common factor of all terms.

Ex.
$$5x^2 + 25 = 5(x^2 + 5)$$

Ex. $5x^2 + 25 = 5(x^2 + 5)$ Make sure to pull out the common factor = 5

When factoring polynomials, follow this rule:

$$x^{2} - 9x + 14 = (x-?) (x-?)$$
 Where $ab = E = 14$
 $x^{2} - Dx + E = (x-a) (x-b)$ $a + b = D = 9$
 $(x-?)(x-?) = (x-2) (x-7) -2x-7 = 14$ $-2+-7 = -9$

Directions: Factor the following polynomials.

1.
$$x^2 - x - 6 =$$

2.
$$x^2 - 6x + 9 =$$

3.
$$x^2 - 7x - 18 =$$

4.
$$3r^3 - 6r^2 - 45r =$$

5.
$$2xy^2 - 54xy + 100x =$$

Rational Expressions

When simplifying rational expressions, factor the numerator and the denominator to cancel out like terms. For example:

$$\frac{4x+6}{2y}$$
 You can factor out a 2 from each term on the top and the bottom. $\frac{2(x+3)}{2(y)}$

Now you can cancel out the two from the numerator and denominator because

$$2 \div 2 = 1$$
 So your answer is: $\frac{x+3}{y}$

Directions: Simplify the following rational expressions to lowest terms.

$$1. \frac{x^2 - 2x - 8}{x^2 - 9x + 20} = \underline{\hspace{1cm}}$$

$$2. \frac{x^2 - 25}{5x - x^2} = \underline{\hspace{1cm}}$$

$$3. \frac{x^3 + 7x^2 + 12x}{x^2 - 9} = \underline{\hspace{1cm}}$$

$$4. \frac{8-3y}{3y^2-2y-16} = \underline{\hspace{1cm}}$$

Linear Equations in One Variable

When given an equation with a variable, you must solve for x. In order to do this, you must get x by itself on one side of the equation. To eliminate the terms around x, do the opposite operation to each side of the equation. See example:

$$3x - 5 = 10$$
 First by adding 5 to both sides (+ is opposite of -)

$$3x-5+5=10+5$$

$$3x = 15$$
 Now divide each side by 3 (÷ is the opposite of x)

$$x = 5$$
 Your answer is 5

Directions: In the following equations, solve for x.

- 1. The sum of 3 consecutive integers is 24. What is the product of these three integers?
- 2. If 15 x 3 = 102, then x = ? _____
- 3. 1-2[4-(3x-5)=3x+1] What is x?
- 4. 12x + 4 [(3 x) (5x + 7)] = 13 What is x?
- 5. $20 \{[3x (x 1)] 5x\} = 0$ What is x?

Linear Equations Solving for One Variable

When solving for a variable in a problem that has multiple variables, you need to isolate the one you're looking for by itself. See the following example solving for x:

$$3x + 5y = ax + 2y$$

Get like terms on the same side of the equation

$$3x - ax = 2y - 5y$$

Factor out the x and combine like terms

$$x(3-a) = -3y$$

Get x by itself by dividing out (3-a)

$$X = \frac{-3y}{3-a}$$

Directions: Solve for a in the following equations.

$$1. \ \frac{2ax}{3c} = \frac{y}{m}$$

1.
$$\frac{2ax}{3c} = \frac{y}{m}$$
 $a =$ ______

2.
$$2cy + 4d = 3ax - 4b$$

2.
$$2cy + 4d = 3ax - 4b$$
 $a =$

3.
$$ax + 3a = bx + 7c$$

3.
$$ax + 3a = bx + 7c$$
 $a =$

4.
$$3(x-a) = 2a - x - \frac{b-x}{c}$$
 $a =$

5.
$$a(x+2) = \pi - cy$$

5.
$$a(x+2) = \pi - cy$$
 $a =$ _____

Linear Equations in Two Variables

Slope: rise/run

The rate at which a line increases or decreases

y-intercept:

Where the line crosses the y-axis

Example: 4x + 3y = 9

Put into y = mx + b form

Slope = m - y-intercept = b

$$4x + 3y = 9$$

Get y by itself

$$3y = 9 - 4x$$

Divide each side by 3

$$y = \frac{-4}{3}x + 3$$

 $y = \frac{-4}{3}x + 3$ slope = $\frac{-4}{3}$ and the y-intercept = 3

Directions: Solve the problems below using what you know about slope/y-intecept

1. What is the slope of the following equation: 5v + 4x = -15

2. On the cooridante plane, line EF passes through points (-5, -3) and (1, 4). What is the slope of line EF?

3. Bob went to the store and bought 3 apples and 2 bananas which cost a total of \$7.00. Then Mary bought 2 apples and 2 bananas which cost a total of \$5.50. How much does each apple cost?

4. Write an equation that corresponds with the data below:

X	1	2	3	4	5	. 6
Υ	9	11	13	15	17	19

Exponents and Radicals

When multiplying radicals, the rule is that: $\sqrt{a} \cdot \sqrt{b} = \sqrt{ab}$

To simplify a radical, find the factors of the radical to find a perfect square:

$$\sqrt{18} = \sqrt{9} \cdot \sqrt{2} = 3\sqrt{2}$$
 9 is a perfect square

For exponents: $a^3(2a^5) = 2a^8$ $(a^4)^3 = a^{12}$

$$a^3(2a^5) = 2a^8$$

$$(a^4)^3 = a^{12}$$

$$\frac{1}{x-1} = x^1$$

$$\frac{1}{x^{-1}} = x^1$$
 $x^{-2} = \frac{1}{x^2}$

Directions: Simplify the expressions below. Leave answers in radical form where it applies.

1.
$$\sqrt{18x^4y^3} \cdot \sqrt{8x^3y^6} =$$

2.
$$\sqrt{6a^2b^7} \cdot \sqrt{8ab} =$$

$$3. \ \frac{(3x^2y)^{-1}}{2xy^5} = \underline{\hspace{1cm}}$$

$$4. \ \frac{3x^{-2}y^3c^5}{x^{-3}y^7c^2} = \underline{\hspace{1cm}}$$

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Functions

Function: An equation that every x value only corresponds to one y value.

$$f(x) = 3x + 2$$

is the same as

$$y = 3x+2$$

To find f(-3), plug -3 in wherever you see an "x" in the equation.

$$f(-3) = 3(-3) + 2 \longrightarrow f(-3) = -9 + 2 \longrightarrow f(-3) = -7 -7 = your answer$$

$$f(-3) = -7$$

Directions: Solve the problems below using the given equation.

1. Let
$$f(x) = 5x^2 - 3$$

1. Let $f(x) = 5x^2 - 3$ Evaluate the following.

2. Let
$$f(x) = 9x + 2$$
 Let $g(x) = x^2 - 2$

Let
$$g(x) = x^2 - 2$$

Evaluate the following.

$$f. g(f(x)) =$$

3. Let
$$f(x) = 6x^2$$

3. Let
$$f(x) = 6x^2$$
 Let $g(x) = 15x - 10$

Evaluate the following.

$$f. g(f(x)) =$$

Math Placement Pre-Test

3.
$$x^2(x^3)^4 =$$

$$4. \frac{-6}{-15} + \frac{3}{5} = \underline{\hspace{1cm}}$$

$$5.\frac{9}{14} \div \frac{5}{21} =$$

$$6.2.3 \times 3.5 =$$

7. If a class received the following set of scores, what would be the mean, median and mode of the scores?

Mean: _____ Mode: ____

$$8. (3x + 1)(2x - 5) = \underline{\hspace{1cm}}$$

9. Factor:
$$x^2 - 15x + 56 =$$

10. Simplify:
$$x - \{2x - 2(1 - x)\} =$$

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